### **Improved TOPSIS decision model for NPP emergencies**

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**Abstract** In this paper, an improved decision model is developed for its use as a tool to respond to emergencies at nuclear power plants. Given the complexity of multi-attribute emergency decision-making on nuclear accident, the improved TOPSIS method is used to build a decision-making model that integrates subjective weight and objective weight of each evaluation index. A comparison between the results of this new model and two traditional methods of fuzzy hierarchy analysis method and weighted analysis method demonstrates that the improved TOPSIS model has a better evaluation effect.

Key words Nuclear emergency, TOPSIS method, Multi-attribute decision-making, Weighted analysis method

#### 1 Introduction

The system of nuclear emergency response decisionmaking aims to provide an effective and reasonable in-time precaution plan when a severe accident occurs to a nuclear power plant (NPP). In the whole decisionmaking process, optimization analysis is an essential<sup>[1,2]</sup>, but this is difficult, too, because in different phases of the nuclear accident, protective measures and criteria that must be taken into account in evaluating strategies change over the time. In order to find a better alternative, it is necessary to make comprehensive evaluations based on multiple indexes like health effects and cost-related issues<sup>[3]</sup>, which differ from each other in dimension as well as in value. Some indexes are cost-oriented, while others are benefit-oriented. These complexities make a nuclear emergency decision-making a fuzzy matter.

Multi-attribute decision-making is to choose one plan from several candidates after taking into account different evaluation indexes. This method has been successfully applied in economy, management, engineering, defense, and so on. The related theories and methods can be categorized as follows: simple weighted analysis method, hierarchy weighted analysis method, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method, fuzzy theory and grey system method<sup>[4]</sup>. In this paper, an improved TOPSIS method is used to build a decision-making model, and the results are compared with the fuzzy and hierarchy decision models, which have been tried in nuclear emergency response decision-making<sup>[5,6]</sup>.

In system engineering, TOPSIS method is a frequently-used technique to work out multi-attribute issues. Its application in treatment of pollution source and economic benefit evaluation has gained satisfying achievements in recent years<sup>[7]</sup>. By building both ideal and negative ideal solutions for a multi-index problem, the method makes evaluations for every feasible plan with two references: approaching ideal solution and away from negative ideal solution. Generally, the basic modeling process can be described as follows. First, an evaluating matrix X is built and a standardization processing is performed to get a standardized matrix Y. Next, matrix Y is combine with index weight vector  $\omega = (\omega_1, \omega_1, \Lambda, \omega_n)$  to get weighted standardized matrix V and thus the ideal and negative ideal solutions for V. Finally, each plan's distance and relative closeness to ideal solution is calculated to prioritize the plans according to their relative closeness. However, this is

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just the traditional way of using TOPSIS method, and its main weakness is that it is fairly subjective, because of the pre-determined weight. The improved TOPSIS method combines the objective weight obtained from decision-making matrix with subjective weight calculated from subjective weighting methods of analytic hierarchy process (AHP) and hierarchy analysis. In this way, more accurate and objective evaluations can be available.

#### 2 Improved TOPSIS model

#### 2.1 Analysis and selection of evaluating indexes

To establish a comprehensive and objective evaluation system for nuclear accident emergency, the selection of evaluation indexes is of the utmost importance. In early phases (hours and days) of a nuclear accident, decision makers are concerned with early-phase countermeasures and criteria such as health effects. In later phases (months and years), more time is allowed to formulate strategies such as combinations of agricultural countermeasures and balancing the short and long-term health effects with the cost<sup>[3]</sup>.

In order to get an optimized strategy, in addition to health and economic influence, other factors such as politics, society and public psychology are considered. For the convenience of quantitative processing, and for making sure that the evaluation is comprehensible and objective, the following indexes can be chosen as evaluating indexes: (1) health risk, the lower, the better; (2) avertable collective dose, the more, the better; (3) avertable individual dose, the more, the better; (4) direct expense, the less, the better; and (5) the expense caused by damage lost in implementing precautions, the less, the better<sup>[8]</sup>.

# 2.2 Evaluation matrix and standardization processing

Suppose there are *m* decision-making plans, and each plan has *n* evaluation indexes, then a evaluation matrix can be built:  $X = (x_{ij})_{m \times n}$ :

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_{11} & \boldsymbol{x}_{12} & \cdots & \boldsymbol{x}_{1m} \\ \boldsymbol{x}_{21} & \boldsymbol{x}_{22} & \cdots & \boldsymbol{x}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \boldsymbol{x}_{m1} & \boldsymbol{x}_{m2} & \cdots & \boldsymbol{x}_{mn} \end{bmatrix}$$
(1)

An index differs from each other in dimension, variation range and value. To avoid the complexity, the general vector normalization method can be used. And indexes in different dimensions can be processed to standardized dimensionless indexes, which are used to build a standardized evaluation matrix  $\mathbf{Y} = (y_{ij})_{m \times n}$ :

$$\boldsymbol{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix}$$
(2)

where

$$y_{ij} = x_{ij} / \sqrt{\sum_{k=1}^{m} \chi_{kj}^2}$$
  $i = 1, 2, \cdots, m; j = 1, 2, \cdots, n$ 

# 2.3 Determination of index weight and weighted standardized matrix

Each of the *n* indexes takes a different level of importance in decision-making, so it is necessary to give them different weights. This is usually done in two groups of approaches. Group 1 is of objective weighted methods with the source information coming from real statistics, including mainly comprehensive index method, entropy weight method, efficacy grading method and principal component analysis. Group 2, of subjective weighted methods with the source information from expert consultation, includes mainly AHP method and fuzzy comprehensive evaluation method<sup>[9]</sup>. The objective weight and subjective weight are calculated.

2.3.1 Determination of objective weight

In determining the objective weight, the information indicated by the standardized evaluation matrix Y can be used. With the minimum value of the weight-square of plan distances to ideal solution as evaluation basis, an optimization model can be established<sup>[4]</sup>.

$$\min = \sum_{i=1}^{m} d_i^2$$

$$y_j^* = \begin{cases} \max_{1 \le i \le m} y_{ij} & i \text{ represents benefit - oriented attribute} \\ \min_{1 \le i \le m} y_{ij} & i \text{ represents cost - oriented attribute} \end{cases}$$

$$d_i^2 = \sum_{j=1}^{n} \left( y_{ij} \lambda_j - y_j^* \lambda_j \right)^2 \quad i = 1, 2, \cdots, m$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j > 0 \quad j = 1, 2, \cdots, n$$

$$(3)$$

where  $d_i$  is the plan distance to ideal solution,  $\lambda_j$  is the objective weight to evaluate index j, and  $y_j^*$  is the ideal solution of attribute j.

Then with the help of the software of LINGO, the objective weight can be calculated with this model. **2.3.2** Determination of subjective weight

Hierarchy analysis method, as one of the most commonly used subjective weighted methods, combines quantitative and qualitative analysis together. By making couple comparisons between each factor, it builds a judgment matrix, whose maximum feature root can direct to a corresponding eigenvector, which can be used as the subjective weight.

**2.3.3** Comprehensive weight and establishment of the weighted standardized matrix

By applying weight-vector synthesis method to normalize objective and subjective weights, one obtains comprehensive weight  $\omega_i$  of attribute indexes:

$$\omega_j = \lambda_j \mu_j / \sum_{j=1}^n \lambda_j \mu_j \qquad 1 \le j \le n$$
(4)

So the weighted standardized matrix V should be:

$$V = \left( v_{ij} \right)_{m \times n} = \left( \omega_j y_{ij} \right)_{m \times n}$$
(5)

# 2.4 Determination of ideal and negative ideal solutions of matrix *Y*

Ideal solution  $v_j^+$  and negative ideal solution  $v_j^-$  is determined as follows:

$$v_j^+ = \begin{cases} \max_{1 \le j \le m} v_{ij}, & i \text{ represents benefit - oriented attribute} \\ \min_{1 \le j \le m} v_{ij}, & i \text{ represents cost - oriented attribute} \end{cases}$$
(6)

$$v_j^- = \begin{cases} \max_{\substack{1 \le i \le m \\ min \le m}} v_{ij}, & i \text{ represents benefit - oriented attribute} \\ min \sum_{\substack{i \le m \\ min \le m}} v_{ij}, & i \text{ represents cost - oriented attribute} \end{cases}$$
(7)

### 2.5 Plan distance to ideal and negative ideal solutions

The plan distance to ideal solution,  $S^+$ , and to negative ideal solution,  $S^-$ , can be calculated by Eqs.(8) and (9).

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2} \qquad i = 1, 2, \cdots, m$$
(8)

$$S_{i}^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{j}^{-})^{2}} \qquad i = 1, 2, \cdots, m$$
(9)

#### 2.6 The relative closeness to ideal solution

Prioritize all the plans according to their relative closeness. The value of  $C_i$  in Eq.(10) shows how close plan *i* is to the optimization. The more, the better.

$$C_{i} = \frac{S_{i}^{-}}{S_{i}^{-} + S_{i}^{+}} \qquad i = 1, 2, \cdots, m$$
(10)

### **3 Emergency decision-making plan** evaluation of an NPP<sup>[1]</sup>

#### 3.1 Evaluation on the plan

In the mid and late stages of a nuclear emergency, multiple measures should be taken into consideration, such as decontamination, food and water control, replacement (if the avertable dose in the first month exceeds  $10^{-2} \text{ J} \cdot \text{kg}^{-1}$ , and in a late month of the year the dose is less than  $10^{-2} \text{ J} \cdot \text{kg}^{-1}$ , residents can return), and permanent resettlement (if the avertable dose is still over  $10^{-2} \text{ J} \cdot \text{kg}^{-1}$  in the  $13^{\text{th}}$  month after replacement). Whether a measure can be applied should depend on the situation. Take a residential area 84 km away from a nuclear power plant, with a population of 3,660 people for example, the detailed parameters are listed in Table 1<sup>[11]</sup>. According to WASH-1400, the accident source term belongs to the 8<sup>th</sup> group (PWR8).

 Table 1
 Accident and environmental parameters

Parameters	Values
Accident source term	PWR8
Wind direction	East wind
Wind velocity / $m \cdot s^{-1}$	1.5
$Rainfall \ / \ mm \cdot h^{-1}$	1

By calculating the estimated and avertable doses in this accident, the following plans can be taken:

Plan A, decontamination;

Plan B, residence replacement (the 12<sup>th</sup> month after the accident);

Plan C, decontamination plus replacement (return in the forth month); and

#### Plan D, permanent resettlement.

The evaluation parameter values are listed in Table 2.

Precaution Strategy	Health effect			Non-health effect	
	Health risk /CNY	ACD* /mSv	Individual dose/mSv	Direct expense /CNY	Damage Lost /CNY
Decontamination	1, 014, 000	10,984,520	9,898	486,000	366, 000
Replacement	1, 762, 000	3,779,337	3,412	12, 700, 000	12, 176, 000
Dec. + Rep.	734, 000	11,127,867	10,028	7, 257, 000	4, 758, 000
Permanent resettlement	1, 764, 000	12,205,023	10,998	146, 400, 000	131, 760, 000

 Table 2
 Parameters for emergency protection strategy<sup>[1]</sup>

\*ACD, avertable collective dose

According to Table 2, a judgment matrix is provided as follows:

	1014	10984520	9898	48.6	36.6
v	1762	3779337	3412	1270	1317.6
$\mathbf{A}_{4\times 5} =$	73.4	11127867	10028	725.7	475.8
	1764	12205023	10098	14640	13176

Combining and optimizing Eq.(3), calculation result shows that an optimal solution can be worked out when the objective weights of the five indexes are:  $\lambda = [0.2128 \quad 0.3328 \quad 0.3333 \quad 0.0605 \quad 0.0605]$ 

With the fuzzy scale matrix method in Ref.[1], all subjective weights can be determined as follows:

The weight vector of health effect and non-health effect is  $\mu_1^{(\rho)} = [0.65 \quad 0.35]$ .

The weight vector of the three indexes to which health effect belongs is  $\mu_2^{(\rho)} = [0.47 \ 0.29 \ 0.24]$ .

The weight vector of the two indexes to which non-health effect belongs is  $\mu_3^{(\omega)} = [0.65 \quad 0.35]$ .

Then, the weight of each index to which health  $(\mu_2)$  and non-health effect  $(\mu_3)$  belong should be resoundingly multiplied by the weight of health (0.65) and non-health (0.35). Thus, subjective weight vector of the five indexes is  $\mu^{(\rho)}=[0.3055 \ 0.1885 \ 0.1560 \ 0.2275 \ 0.1225]$ .

Based on Eq. (4), the comprehensive weight of each evaluation index is  $\omega^{(p)} = [0.3236 \ 0.3122 \ 0.2588 \ 0.0685 \ 0.0369].$ 

According to Eq.(5), a weighted standardized matrix can be obtained. From Eqs.(6) and (7), the ideal  $v^+$  and negative ideal solution  $v^-$  are;

 $v^+ = [0.0851 \quad 0.1887 \quad 0.1584 \quad 0.0002 \quad 0.0001]$  $v^- = [0.2047 \quad 0.0584 \quad 0.0485 \quad 0.0682 \quad 0.0367]$ 

Combing Eqs.(8), (9) and (10), the relative closeness of each strategy to ideal and negative ideal solutions is:

 $C = [0.8021 \quad 0.2540 \quad 0.9030 \quad 0.5430]$ 

In accordance with "the more, the better" principal, it can be concluded that the priority order of the four feasible precaution plans is Plan C, Plan A, Plan D and Plan B.

## **3.2** Comparing the improved TOPSIS method to other methods

To testify the accuracy and effectiveness of the results obtained from the improved TOPSIS model, a comparison is made between this method and the traditional methods of the weighted analysis method and fuzzy hierarchy analysis method.

**3.2.1** Weighted analysis method

The process of using weighted analysis method to deal with the same example is as follows:

(1) Normalize the judgment matrix  $X_{4\times 5}$  in Section 3.1, matrix  $Y_{4\times 5}$  can be obtained as

$$y_{ij} = x_{ij} / \max_i x_{ij}$$

(2) Forward processing the comprehensive weight  $\omega^{(\rho)}$  obtained in Section 3.1, one obtains the weight vector  $\vec{\alpha} = [-0.3236 \quad 0.3122 \quad 0.2588 \quad -0.0685 \quad -0.0369].$ 

(3) Multiply  $Y_{4\times 5}$  by transposed vector  $\vec{\alpha}$  to obtain the weighted comprehensive vector  $\vec{\beta} = [0.3276 - 0.1556 0.3813 0.1420]$ .

3.2.2 Comparison between different methods

For the priority order of the plans, using the improved TOPSIS method can come to the same conclusion as demonstrated by weighted analysis method and fuzzy hierarchy method in Ref.[1]. However, in judging the superiority and inferiority of the plans, the three methods differ in quantitative evaluation. According to Ref.[1], in fuzzy hierarchy decision-making model, each plan's relative membership degree vector is  $\mu_A = [0.9770 \quad 0.4500 \quad 0.9910 \quad 0.5470]$ 

In order to put the results of the three methods under the same scale for comparison, it is necessary to process related data with Steps I and II.

Step I: 
$$c'(i) = c(i) / (\max_{i} c(i) - \min_{i} c(i))$$

The method called "scaling" can be used on  $\beta$ (*i*) and  $\mu_A(i)$ . After processing,  $\beta'(i)$  and  $\mu_A'(i)$  are obtained. This is to ensure that the three groups of evaluation data are consistent in length, which means that the difference between the maximum and minimum value of each group is "1".

Step II: Respectively subtract [min  $\beta'(i)$ -min c'(i)] and [min  $\mu_A'(i)$ -min c'(i)] from  $\beta'(i)$  and  $\mu_A'(i)$ . This is to ensure the consistence of the positions for the three groups of evaluation data by way of translation, making each group have the same minimum value. Table 3 shows the processed data of each method for Plans A, B, C and D.

 Table 3
 Evaluation results of the three methods after processing\*

Methods	Plan A	Plan D
Fuzzy hierarchy analysis	1.365	0.571
Improved TOPSIS	1.236	0.837
Weighted analysis	1.291	0.946

\* The evaluation results of Plans B and C are 0.391 and 1.391, respectively, for all the three methods.

To determine the superiority of the methods, the concept of "relative resolution" is used. It can elaborate each method's ability to differentiate every possible plan. Take TOPSIS method for instance, the relative resolution that distinguishes Plan i from Plan j is |c'(i) - c'(j)|. The higher the relative resolution is, the subtler the evaluation can be. According the priority order, the resolution of neighboring plans is listed in Table 4.

 Table 4
 Relative resolution comparison of the three methods

Methods*	Plans C and A	Plans A and D	Plans D and B
IT	0.1555	0.3992	0.4453
FHA	0.0259	0.7948	0.1793
WA	0.1001	0.3457	0.5543
Average	0.0938	0.5132	0.3930

\*IT is improved TOPSIS, FHA is fuzzy hierarchy analysis and Wa is weighted analysis.

From Table 4, it can be seen that it is the most difficult to differentiate Plan C from Plan A, whereas the improved TOPSIS method can do a relatively better job in identifying the superior one. Also, the TOPSIS method has the same effectiveness in distinguishing Plan A, Plan D and Plan B. Therefore, it can be concluded that the improved TOPSIS method has a better evaluation effect than fuzzy hierarchy analysis method and weighted analysis method.

#### 4 Conclusion

The decision-making on nuclear emergency is a complicated process that requires comprehensive considerations based on many factors. Picking a group of reasonable evaluating indexes from many factors is far from enough. The objective and subjective weights of every index should be taken into account, too. The TOPSIS method used in this paper, by calculating each plan's relative closeness to ideal solution, can be a reasonable optimization issue in selecting plans, with a better evaluation effect.

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