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Determination of fuel burnup distribution of a research reactor based on measurements at subcritical conditions

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Abstract This paper presents the determination of the fuel burnup distribution of the Dalat nuclear research reactor (DNRR) using a method of measurements at subcritical conditions. The method is based on the assumption of linear dependence of the reactivity on the burnup of fuel bundles and the measurements at subcritical conditions. The measurements were taken for seven selected fuel bundles in two different measuring sequences. The measured burnup values have also been compared with the calculations for verifying the method and the measurement procedure. The results obtained with the three detectors have a good agreement with each other with a discrepancy less than 1.0%. The errors of the measured burnup values are within 6%. Comparison between the calculated and measured burnup values shows that the discrepancy of the *C*/*E* ratio is within 9% compared to unity. The results indicate that the method of measurements at subcritical conditions could be well applied to determine the relative burnup distribution of the DNRR.

Keywords Burnup distribution · Subcritical measurement · Research reactor · DNRR

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1 Introduction

Determination of fuel burnup is one of the important tasks to improve the performance, safety, and economics of a research reactor. Various methods are available for burnup measurement of fuel bundles in research reactors based on destructive and non-destructive techniques [1]. Destructive methods such as chemical techniques could determine accurately the depletion of fissile materials in irradiated fuel bundles and thus can determine accurately the absolute burnup values of the spent fuel [2, 3]. However, the destructive methods are usually complicated and expensive [3, 4]. The advantage of non-destructive methods over the destructive ones is that the measurements can be taken without affecting the fuel integrity [5]. Three common non-destructive methods for determining fuel burnup are (1) reactor physics calculations, (2) measurement of reactivity effects, and (3) gamma-ray spectrometry [1]. The method based on reactor physics calculations is powerful since the calculations can be performed without any restriction. Therefore, reactor physics calculations are usually used to support and verify the results obtained from other measurement methods or vice versa.

Gamma-ray spectrometry is an effective non-destructive technique based on the measurements of the activity of specific fission products, e.g ^{134}Cs and ^{137}Cs [4–9]. The relative distribution of the fission products in a fuel element depends on the irradiation history, fuel type, and cooling time. The measured ^{134}Cs/^{137}Cs activity ratio distributions were used to estimate the fuel element averaged burnup [4]. However, in the gamma-ray spectrometry, the fuel elements need to be transferred to a hot cell for the measurements [4]. This is a limitation when a large number of

fuel elements need to be measured or the fuel elements are reloaded in the next cycle.

Fuel burnup measurement methods based on the reactivity effects take an advantage due to short measuring time. These methods can be classified as the measurements under critical or subcritical conditions [10, 11]. In a past work, a method for determining the relative fuel burnup distribution based on the measurements at subcritical conditions was proposed [11]. The method relies on an assumption of a linear relationship between fuel bundle reactivity and its burnup. The assumption may not be valid for a fuel bundle loaded with burnable poisons; however, for a research reactor, this assumption is valid. In the DNRR core with HEU fuel bundles, the linear dependence of reactivity on fuel burnup is valid up to a 30% loss of ^{235}U [11]. In another work by Pinem et al. [12], the same method was applied for determining the relative burnup distribution of the LEU silicide RSG GAS research reactor. The burnup of fuel elements reaches up to 50% loss of ^{235}U. It means that the range of burnup is high for the assumption of linear dependence of the reactivity.

The advantage of the method based on subcritical conditions is that the measurement procedure is simple and inexpensive. However, this method is not able to determine the absolute burnup, but the relative burnup distribution. The method could contribute as an alternative option for verifying other methods or techniques of burnup determination. Application and verification of this method have been performed for determining the relative burnup distribution of research reactors [11, 12]. In recent burnup measurements for the LEU silicide RSG GAS reactor using the method of measurements at subcritical conditions, the relative burnup values of 22 fuel elements with the burnup of about 20–53% loss of ^{235}U were measured [12]. A good agreement between the measurements and the calculations was found with the uncertainty within 8%.

In the past works [11], the burnup measurements were taken in a single sequence, i.e. measurements with the same subcritical configuration. However, in practice, it is not usually possible to measure the burnup distribution of a whole core or a large number of fuel elements in a single sequence since a large number of the fuel elements are needed to construct a subcritical configuration for the measurements. In this case, successive sequences of the measurements with different core configurations could be applied. Pinem et al. conducted the measurement in multi-successive sequences, but the detailed procedure has not been discussed [12]. In the present work, the method of measurements at subcritical conditions is revisited and applied to determine the relative fuel burnup distribution of the DNRR. This is an extension of the previous works [11, 12] to demonstrate and verify the measurement procedure of relative burnup distribution in multi-successive sequences. The burnup measurements have been taken for seven selected fuel bundles in a HEU core configuration of the DNRR. The measurement procedure was performed in two different sequences using three fission counters located in the water reflection region. Calculations of the burnup distribution have also been performed using the WIMSD/CITATION codes and compared to the measurement results.

2 Methods

2.1 Burnup measurements at subcritical conditions

This section summarizes the derivation of the burnup measurement method at subcritical conditions [11, 13]. The method is based on the assumption of linear dependence of the reactivity of the fuel bundles on burnup. When a reactor is operated at a subcritical (close to critical) state with a neutron source, the neutron density of the reactor reaches a stable value N [13]:

$$N = -N_{\mathrm{src}}/\delta k, \quad \delta k < 0, \tag{1}$$

where N_{src} is the neutron density of the source, $\delta k = (k-1)$ is the subcriticality of the reactor, and k is the effective multiplication factor. The reactor state is constructed so that a fresh fuel bundle is inserted into a specific location in the core and the reactor still remains at subcriticality, δk_0. This means that the reactivity insertion of the fresh fuel bundle is not enough for the reactor to get criticality. The stable neutron density of the reactor in this case is denoted as N_0 and is calculated as:

$$N_0 = -N_{\mathrm{src}}/\delta k_0, \quad \delta k_0 < 0. \tag{2}$$

If the fresh fuel bundle is withdrawn from the core and no fuel bundle is inserted into the specific location, i.e. the location is free, the stable neutron density in this case of the reactor is denoted as N_{free} and is calculated as:

$$N_{\mathrm{free}} = -N_{\mathrm{src}}/\delta k_{\mathrm{free}} = -N_{\mathrm{src}}/(\delta k_0 - \rho_0), \quad \rho_0 > 0, \tag{3}$$

where $\delta k_{\mathrm{free}} = (\delta k_0 - \rho_0)$ is the subcriticality of the reactor when the fresh fuel bundle is withdrawn. ρ_0 is the reactivity of the fresh fuel bundle.

Now assuming that there are G fuel bundles needed to measure their burnup by successively inserting into the same location in the reactor core. The neutron density at stable subcritical states with the insertion of fuel bundle i is

$$N_i = -N_{\text{src}}/\delta k_i = -N_{\text{src}}/(\delta k_0 - \rho_0 + \rho_i), \quad i = 1, \ldots, G, \tag{4}$$

where N_i and δk_i are the neutron density and the subcriticality of the reactor associated with the insertion of fuel bundle i, respectively. ρ_i ($\rho_i > 0$) is the insertion reactivity associated with the insertion of fuel bundle i. From Eqs. (2)–(4), one obtains the relationship between the stable neutron densities and the reactivity as follows:

$$\frac{N_{\text{free}}}{N_0} = 1 + \frac{\rho_0}{\delta k_{\text{free}}}, \tag{5}$$

and

$$\frac{N_{\text{free}}}{N_i} = 1 + \frac{\rho_i}{\delta k_{\text{free}}}. \tag{6}$$

From Eqs. (5) and (6), the relationship between the insertion reactivity of the fresh fuel bundle, ρ_0, and the burnt fuel bundle, ρ_i, is obtained

$$\rho_i = c_i \rho_0 \quad i = 1, \ldots, G, \tag{7}$$

where

$$c_i = \frac{N_0}{N_0 - N_{\text{free}}} \frac{N_i - N_{\text{free}}}{N_i}. \tag{8}$$

As long as the assumption of the linear dependence of the reactivity on fuel burnup is valid, the relationship between ρ_i and ρ_0 can be written as

$$\rho_i = \rho_0 + b\text{BU}_i \quad i = 1, \ldots, G, \tag{9}$$

where BU_i is the absolute burnup of fuel bundle i and b is a constant. Then, the relative burnup of fuel bundle i can be obtained by normalizing the absolute burnup values to an average burnup value of all fuel bundles as follows:

$$BU_i^* = \frac{\text{BU}_i}{\overline{\text{BU}}} = \frac{(c_i - 1)}{\frac{1}{G}\sum_{j=1}^{G}(c_j - 1)} \quad i = 1, \ldots, G, \tag{10}$$

where $\overline{BU} = \frac{1}{G}\sum_{j=1}^{G}(c_j - 1)$ is the average burnup of the G fuel bundles. It can be seen from Eq. (10) that the relative burnup of fuel bundle i can be determined only by measuring the neutron densities N_0, N_{free}, and N_i.

Here, it is assumed that the reactor core is loaded by the same fuel type, i.e. the same material composition, enrichment, so that Eq. (9) can represent for all fuel bundles. Thus, the coefficient b is eliminated in Eq. (10). In the case the reactor core is loaded with different fuel types, e.g. different material compositions, enrichment, Eq. (9) cannot represent for all fuel bundles. In order to extend the application of this method, one needs to estimate the coefficient b of all fuel types prior to formulate Eq. (10).

2.2 Renormalization of the relative burnup distribution

Once the measurements of the fuel bundles are taken in a single sequence (i.e. the measurements at the same location in the same core configuration), the relative burnup distribution is sufficiently determined by Eq. (10). However, it is not usually practical to measure the burnup of all fuel bundles in a single sequence since a large number of the fuel bundles are used to construct a subcritical core configuration. Thus, the measurements need to be taken in a number of sequences with different core configurations. In each sequence, the measurements are taken for a number of fuel bundles for obtaining a set of relative burnup values. These relative burnup values may be different in different sequences since they are normalized to the average burnup value of each sequence. Hence, it is necessary to renormalize the relative burnup values for obtaining the relative burnup distribution of the core.

We assume that the reactor core consists of G fuel bundles and the measurements of the burnup of all G fuel bundles are taken in M different sequences. In particular, in sequence 1, the number of fuel bundles to be measured is M_1 ($M_1 < G$). A subcritical core configuration is established by the rest of fuel bundles to measure the relative burnup of the M_1 fuel bundles. As a result, the relative burnup of the M_1 fuel bundles is obtained as

$$BU_i^{1*} = \frac{BU_i}{\overline{BU}^1} = \frac{BU_i}{\frac{1}{M_1}\sum_{j=1}^{M_1} BU_j} \quad i = 1, \ldots, M_1. \tag{11}$$

In Eq. (11), the burnup values are normalized to the average burnup $\overline{BU}^1$ of the M_1 fuel bundles in this sequence for obtaining the relative burnup, BU_i^{1*}.

Similarly, in sequence 2 the measurements were taken for M_2 fuel bundles. One obtains the relative burnup of the M_2 fuel bundles as:

$$BU_i^{2*} = \frac{BU_i}{\overline{BU}^2} = \frac{BU_i}{\frac{1}{M_2}\sum_{j=1}^{M_2} BU_j} \quad i = 1, \ldots, M_2. \tag{12}$$

However, if a certain fuel bundle is measured in both sequence 1 and sequence 2, its relative burnup values in the two sequences are different since they are normalized to different average burnup values as in Eqs. (11) and (12). Thus, one needs to renormalize the relative burnup distribution so that the same fuel bundles measured in several sequences should have identical burnup values. Here, in sequence 2 instead of normalizing to the average burnup value of this sequence as in Eq. (12), it is proposed to normalize to the average burnup of sequence 1 as follows:

$$BU_i^{2*} = \frac{BU_i}{\overline{BU}^1} = \frac{BU_i}{\frac{1}{M_1}\sum_{j=1}^{M_1} BU_j} \quad i = 1, \ldots, M_2. \tag{13}$$

The same normalization (normalized to the average burnup of sequence 1) is applied to all other measuring sequences, i.e. in sequence I with the measurements of the M_I fuel bundles, the relative burnup of the M_I fuel bundles is:

$$BU_i^{I*} = \frac{BU_i}{\overline{BU}^1} = \frac{BU_i}{\frac{1}{M_1}\sum_{j=1}^{M_1} BU_j} \quad i = 1, \ldots, M_I. \tag{14}$$

This is to ensure that all burnup values are normalized in the same way. After all measuring sequences are completed, one obtains the G relative burnup values for all G fuel bundles. The relative burnup distribution in the core can be obtained by the renormalization of the values as follows:

$$BU_i^{**} = \frac{BU_i^*}{\frac{1}{G}\sum_{j=1}^{G} BU_j^*} \quad i = 1, \ldots, G. \tag{15}$$

2.3 The DNRR

The DNRR is a 500-kW pool-type research reactor located in Dalat, Vietnam. In the early 1980s, the DNRR was reconstructed and upgraded from the 250-kW TRIGA Mark-II reactor. The first criticality of the DNRR was achieved on 1 November 1983, and the regular operation with full power was achieved in March 1984. The first core was loaded with 88 high-enriched uranium (HEU) fuel assemblies with an ^{235}U enrichment of 36% [14, 15]. In the framework of the programme on Russian Research Reactor Fuel Return (RRRFR) and the programme on Reduced Enrichment for Research and Test Reactor (RERTR), the DNRR core was partly converted from HEU fuel to low-enriched uranium (LEU) fuel with ^{235}U enrichment of 19.75% in September 2007. Then, the full core conversion to LEU fuel was performed during the period from 24 November 2011 to 13 January 2012.

Figure 1 shows the horizontal cross-section view of the DNRR reactor. The core consists of 121 hexagonal cells including fuel bundles, control rods, beryllium rods, irradiation channels, and beryllium blocks as shown in Fig. 2. The active core has a diameter of about 44.2 cm and a height of 60 cm and is loaded with the Russian VVR-M2 fuel type. The active core is surrounded by a graphite reflector with the thickness of 30.5 cm. The power output of the DNRR reactor is 500 kW. The reactor core is controlled by seven control rods: two safety rods, four shim rods, and one automatic regulating rod. The safety and shim rods are made of boron carbide (B_4C), while the automatic regulating rod is made of stainless steel.

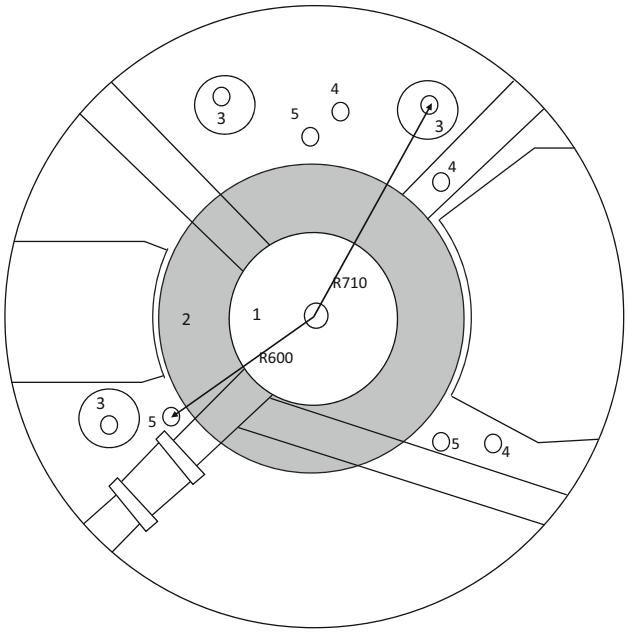

Fig. 1 Horizontal cross-section view of the DNRR research reactor. 1—active core, 2—graphite reflector, 3—intermediate range detectors, 4—source range detectors (SD), and 5—power range detectors

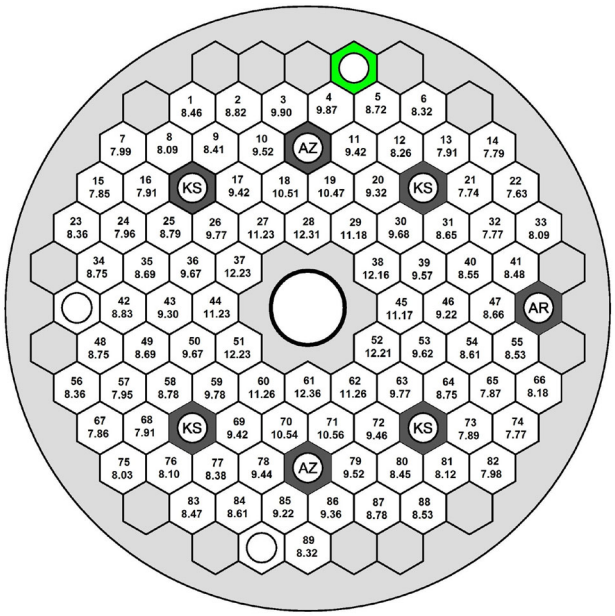

Fig. 2 DNRR core configuration with HEU fuel. AZ—safety rod; KS—shim rod; AR—automatic regulating rod. The data in each block show the order of fuel bundles and the declared absolute burnup in per cent loss of ^{235}U

Figure 3 shows the cross section of the VVR-M2 type HEU fuel bundle, which is made of aluminium uranium alloy cladded in aluminium. The enrichment of ^{235}U in the HEU fuel bundle is 36 wt%. The total ^{235}U mass in the HEU fuel bundle is about 40.2 g distributed in three

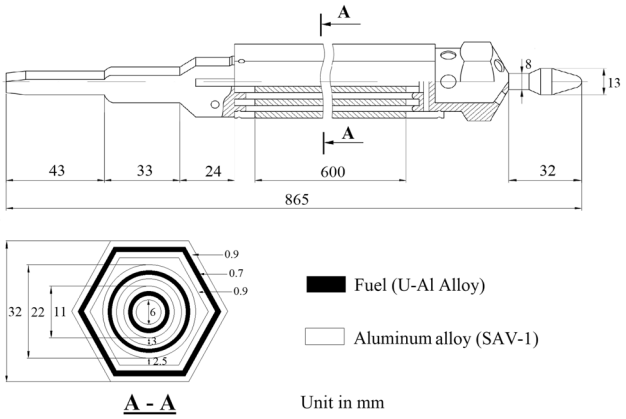

Fig. 3 The VVR-M2 type HEU fuel bundle of the DNRR

coaxial fuel tubes. The outer tube has a hexagonal shape, and the two inner tubes are cylindrical.

2.4 Measurements

For measuring the neutron flux, nine neutron detectors are placed in dry channels inserted in the water reflection region between the graphite reflector and the reactor aluminium tank (see Fig. 1). The detectors measure the neutron flux in three ranges: source range, intermediate range, and power range. The source range and intermediate range detectors are fission chambers of the KNK-15 type, working in pulse mode. The power range detectors are ^{10}B ionization chambers of the KNK-3 type, working in current mode. In the present work, the three source range detectors (SD1, SD2, and SD3) were used in the measurement of the neutron densities for determining the relative burnup distribution. The three source range detectors are located far from the active core, beyond the graphite reflector. Therefore, they are not sensitive to radial change of neutron flux in the active core.

The critical core loaded with 89 HEU fuel bundles was chosen for measuring the relative burnup distribution as shown in Fig. 2. The measurement position was chosen based on the practical arrangement of the DNRR core which set the position (1–4) for the irradiation wet channel. It is also more convenient to fix the position of the irradiation channel during the measurements than to change the measurement positions in each sequence. Furthermore, it is not recommended to choose the position adjacent to shim or safety rods, which are made of neutron absorption material, because the diffusion theory would lead to higher uncertainty. The method itself is based on the point kinetic assumption so that it is independent on the measurement position.

In the present work, we aim to demonstrate and verify the proposed method of measurements in multi-sequences. Seven fuel bundles have been selected for the measurements in two different sequences. In the first sequence, the measurements were taken for four fuel bundles (bundle code: 154, 156, 193, and 194). The locations of the selected fuel bundles in the core are listed in Table 1 corresponding to the locations indicated in Fig. 4. In the second sequence, four fuel bundles with the bundle codes of 154, 160, 189 and 79 were measured. The fuel bundle 154 was selected to measure in both sequences. It is noticed that in order to renormalize the relative burnup distributions obtained in the two sequences, it is required to have at least one fuel bundle measured in the two sequences. The measurement procedure is summarized in Fig. 5.

3 Results and discussion

Calculations of the burnup distribution have also been performed using the WIMSD/CITATION codes and compared to the measurement results [16, 17]. In the calculation model, the triangular meshes were used, in which each hexagonal bundle was divided into six triangular meshes. The fuel bundle pitch is about 3.5 cm, which is small enough to use six meshes for each fuel bundle in the calculations using the CITATION code based on the finite difference method. Correction of the number densities at the boundary meshes of the outer beryllium, the graphite reactor, and some neutron channels was treated to accurately simulate the cylindrical boundary using the CITATION code. Similar correction was applied for full core calculations using the SRAC/CITATION codes in order to

Table 1 Measured relative burnup of the first bundle group using detectors SD1, SD2, and SD3

No.	Bundle code	Location	N_i (cps)	BU_i^*
Measurement with detector SD1				
1	154	5–6	1564.49 ± 50.01	1.312 ± 0.083
2	156	3–4	1703.47 ± 54.40	1.128 ± 0.079
3	193	5–3	1923.65 ± 72.94	0.891 ± 0.080
4	194	7–3	1868.64 ± 65.64	0.945 ± 0.078
Measurement with detector SD2				
1	154	5–6	831.60 ± 30.15	1.322 ± 0.101
2	156	3–4	911.06 ± 36.80	1.129 ± 0.099
3	193	5–3	1032.92 ± 44.14	0.890 ± 0.093
4	194	7–3	1003.07 ± 44.62	0.943 ± 0.097
Measurement with detector SD3				
1	154	5–6	2409.93 ± 75.82	1.303 ± 0.082
2	156	3–4	2612.34 ± 91.21	1.129 ± 0.082
3	193	5–3	2951.24 ± 107.00	0.892 ± 0.078
4	194	7–3	2864.89 ± 98.70	0.947 ± 0.077

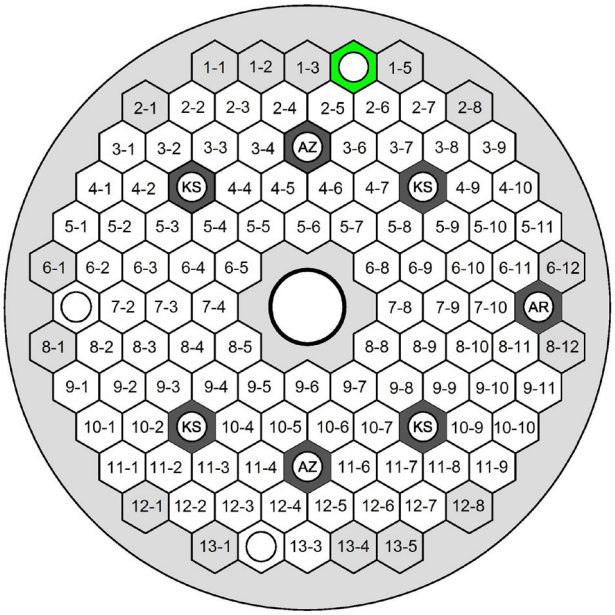

Fig. 4 Subcritical configuration of the DNRR core used for the burnup measurements. The fuel bundle is inserted into the location 1–4 for burnup measurement

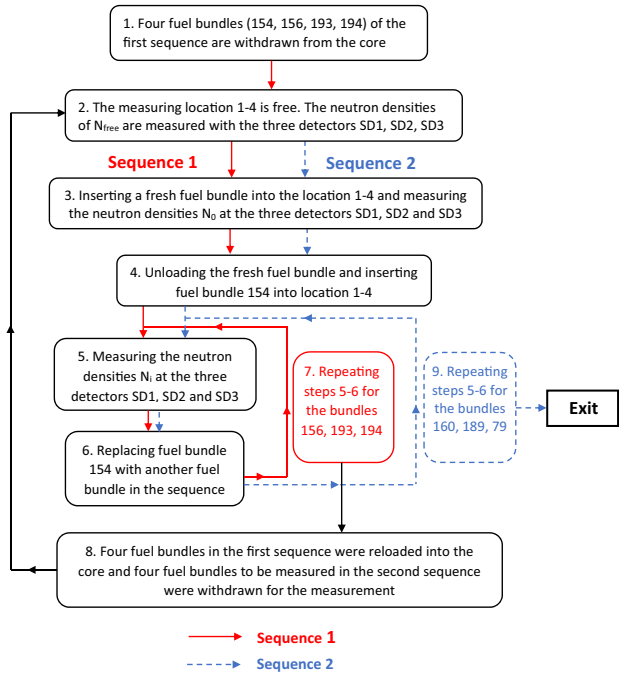

Fig. 5 Measurement procedure of the relative burnup distribution of the seven fuel bundles in two successive sequences

obtain a good agreement with the results obtained in MCNP5 calculations [18].

Table 1 shows the relative burnup distribution of four fuel bundles (bundle codes: 154, 156, 193, and 194) measured in the first sequence using the three source range detectors SD1, SD2, and SD3, respectively. Similarly,

Table 2 Measured relative burnup of the second bundle group using detectors SD1, SD2, and SD3

No.	Bundle code	Location	N_i (cps)	BU_i^*
Measurement with detector SD1				
1	154	5–6	1443.48 ± 44.77	1.185 ± 0.076
2	160	4–6	1630.01 ± 52.54	0.942 ± 0.072
3	189	4–4	1576.00 ± 54.81	1.007 ± 0.075
4	79	2–3	1699.50 ± 57.04	0.866 ± 0.072
Measurement with detector SD2				
1	154	5–6	732.87 ± 27.83	1.190 ± 0.090
2	160	4–6	831.47 ± 33.81	0.943 ± 0.086
3	189	4–4	805.90 ± 31.27	1.001 ± 0.086
4	79	2–3	868.28 ± 35.05	0.866 ± 0.083
Measurement with detector SD3				
1	154	5–6	2117.05 ± 64.81	1.176 ± 0.071
2	160	4–6	2390.71 ± 75.09	0.942 ± 0.067
3	189	4–4	2307.11 ± 71.96	1.007 ± 0.069
4	79	2–3	2481.15 ± 78.41	0.875 ± 0.066

Table 3 Relative fuel burnup obtained from renormalization of the burnup values in sequence 2 so that the measured relative burnup values of fuel bundle 154 in the two sequences are identical

No.	Bundle code	Location	BU_i^*
1	154	5–6	1.312 ± 0.051
2	156	3–4	1.129 ± 0.050
3	193	5–3	0.891 ± 0.048
4	194	7–3	0.945 ± 0.041
5	160	4–6	1.045 ± 0.048
6	189	4–4	1.114 ± 0.049
7	79	2–3	0.963 ± 0.047

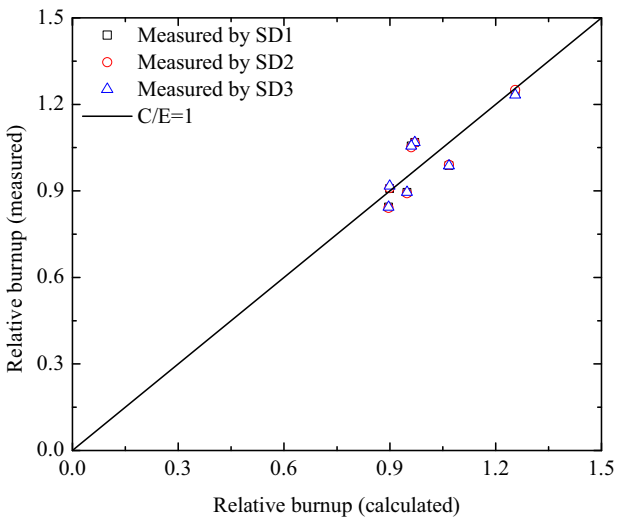

Fig. 6 Comparison among the relative burnup distributions measured by the three detectors

Table 4 Average relative fuel burnup distribution measured by the three detectors in comparison with the calculated values

No.	Bundle code	Location	Relative burnup		*C/E*
			Exp.	Calc.	
1	154	5–6	1.241 ± 0.053	1.256	1.011 ± 0.043
2	156	3–4	1.068 ± 0.051	0.971	0.909 ± 0.043
3	193	5–3	0.843 ± 0.048	0.897	1.064 ± 0.061
4	194	7–3	0.894 ± 0.042	0.949	1.061 ± 0.050
5	160	4–6	0.988 ± 0.049	1.068	1.080 ± 0.054
6	189	4–4	1.054 ± 0.050	0.961	0.912 ± 0.043
7	79	2–3	0.911 ± 0.047	0.900	0.987 ± 0.051

Table 2 shows the relative burnup distribution of four fuel bundles measured in the second sequence (bundle codes: 154, 160, 189, and 79) using the three spectrum detectors SD1, SD2, and SD3, respectively. One can see from Tables 1 and 2 that the errors of the measurement results with each detector are all within 6–10%. Since fuel bundle 154 was measured in both sequences, its burnup values in the two sequences should be renormalized to be identical according to Eq. (12). Table 3 presents the renormalized values of the relative fuel burnup of the seven fuel bundles in the two measuring sequences. It is noticed that the relative burnup values in this table are averaged over the data obtained from the three detectors; thus, the errors (standard deviations) within 6% are smaller than that in Tables 1 and 2. Figure 6 shows the comparison of the relative burnup values of the seven selected fuel bundles measured by the three detectors SD1, SD2, and SD3 after renormalization of the results obtained in the two sequences. It can be seen

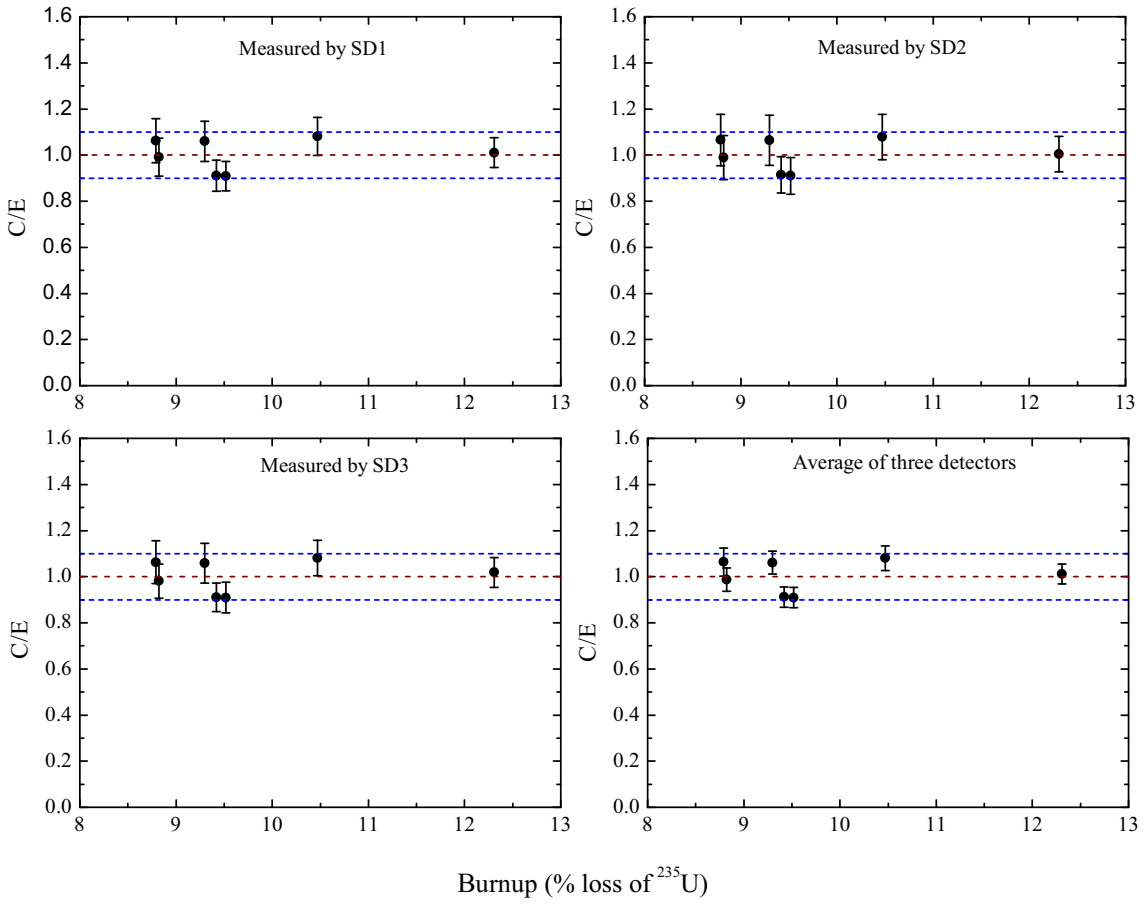

Fig. 7 Comparison between the calculated and measured burnup distribution obtained with the three detectors SD1, SD2, and SD3. The lower right figure shows the *C/E* ratio of the calculation and the average measured value of the three detectors. The error bars are determined by the statistical errors of neutron densities measured by the three detectors

that the relative burnup distributions of all fuel bundles obtained with the three detectors have a good agreement with the deviation less than 1.0%.

Table 4 shows the relative burnup distribution of the seven measured fuel bundles in comparison with the calculation values. The measured burnup values are the average values measured by the three detectors. The measured relative burnup was averaged by the values obtained with the three detectors. It can be seen that the errors of the measured burnup values are within 6%. Comparing the relative burnup distribution of the seven bundles with that derived from the declared burnup in Fig. 2, the discrepancy is less than 10%. Figure 7 and Table 4 depict the *C*/*E* ratio of the relative burnup of the seven measured fuel bundles corresponding to the three detectors and the average of them. One can see that fuel bundles 79 and 154 have a good agreement between the measurements and calculations with the discrepancy of about 2%. The largest discrepancy of the *C*/*E* ratio compared to unity is about 9%. One of the possible reasons for the discrepancy between the calculated and the measured results might be the correction of the fuel bundle geometry in the lattice calculations using the WIMSD code. When comparing the k_{∞} value with that obtained from MCNP5 calculation, the difference is about 180 pcm, which is considerably acceptable [18]. Another reason is that it is also difficult to determine exactly the positions (insertions) of the control rods in the measurement and simulate them accurately. The uncertainty of the determination of control rod insertion is 1 mm, which corresponds to the uncertainty of the reactivity worth of the four shim rods of about 0.028 $, or about 6% of the reactivity worth of a fuel bundle. The agreement between calculations and measurements obtained in this work implies that the method of subcritical measurements in multi-sequences was well applied to determine the relative burnup distribution of the DNRR.

4 Conclusion

Burnup distribution of the Dalat nuclear research reactor has been determined using the method of measurements at subcritical conditions. The measurements were taken for seven selected fuel bundles in two different sequences using the three detectors located outside the graphite reflector. Calculations of the relative burnup distribution have also been carried out using the WIMSD/CITATION codes and compared to the measured results. The measured burnup distribution obtained with the three detectors has a good agreement with the discrepancy within 1.0%. The errors of the measured burnup values are within 6% for all fuel bundles. Comparison between the measured and calculated results shows that the discrepancy of the *C*/*E* ratio is within 9% compared to unity. The agreement confirms the successful application of the subcritical measurement procedure in multi-sequences for burnup determination of the DNRR reactor. Since the measurement procedure at subcritical conditions is simple and inexpensive, it provides an alternative for determining the relative burnup distribution of research reactors and/or for verifying other methods of burnup measurement.

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